Weinberg 2.5

We consider 2 space 1 time symmetries $SO(2,1)$,
then naturally,

$$\eta_{ij} = \begin{pmatrix} -1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

Apply $\Lambda^T \eta \Lambda = \eta$, $\Lambda^a{}_i \eta^{ij} \Lambda^b{}_j = \eta^{ab}$,

write for infinitesmal from unity,

$$\Lambda^a{}_i = \delta^a{}_i + \omega^a{}_i, \quad \text{then}$$

$$\Lambda^a{}_i \eta^{ij} \Lambda^b{}_j = (\delta^a{}_i + \omega^a{}_i)\eta^{ij}(\delta^b{}_j + \omega^b{}_j) = \eta^{ab}$$

$$= \eta^{ab} + \omega^{ba} + \omega^{ab} + O(\omega^2) = \eta^{ab}$$

$$\Rightarrow \boxed{\omega^{ab} + \omega^{ba} = 0} \quad \text{as expected.}$$

For 3×3 ω^{ab}, there are only 3 independent components,
so the transformation $\Lambda^a{}_b$ is parameterized by 3 #s:

$$\Lambda^a{}_b = \Lambda^a{}_b(\theta_1, \theta_2, \theta_3)$$

of course, this is not considering 2 more independent components
from 2-space translation.

U(Lorentz grp element, translation grp element)

$= U(\Lambda, a)$

For Λ, a infinitesmal from being trivial, this is

$$U(1+w, \varepsilon) \simeq 1 + \frac{1}{2} i w_{ab} J^{ab} - i \varepsilon_c P^c$$

for Hermitian J, P.

Since we are in 2 space, P is 2-Dimensional, there would also have to be 2 independent boost generators, so this leaves only 1 independent component for ~~angula~~ rotation, this means the angular momentum is trivial!? As in invariant under Lorentz transformations? (I'm actually surprised and not sure about this).

The momentum of the particle shall transform under ordinary 2D Lorentz trans.

$$\Lambda = \begin{pmatrix} \gamma & & -\gamma\beta \\ & 1 & \\ -\gamma\beta & & \gamma \end{pmatrix}$$

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